DOCUMENT RESUME

ED 210 841 EC 140 641

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TITLE	Story Problem Solving: Implications of Research for Teaching Children with Learning Disabilities. Technical Report # 12.
INSTITUTION	Columbia Univ., New York, N.Y. Research Inst. for the Study of Learning Disabilities.
SPONS AGENCY	Office of Special Education (ED), Washington, D.C.
PUB DATE	80
CONTRACT	300-77-0491
NOTE	31p.
EDRS PRICE	MF01/PC02 Plus Postage.
DESCRIPTORS	Foreign Countries; *Learning Disabilities; Literature Reviews; Logical Thinking; *Mathematics; *Problem Solving; Teaching Methods
IDENTIFIERS	USSR

ABSTRACT

The authors draw upon both Soviet and American literature to present an overview of the components of story problem solving as it relates to remedial instruction for learning disabled students. Factors within the individual problem solver are explored, and strategies children use in solving problems are examined. Particular attention is paid to analytic and intuitive strategies, and to how the child attempts to make the transition from concrete to symbolic problems. The most widely supported instructional principles are reviewed. The phenomenon that children who present oral solutions frequently err when writing the answers is explained. The paper concludes with a review of how studies cited may contribute to an improved methodology for researching story problem solving. (Author/SB)

ED210841

RESEARCH INSTITUTE FOR THE STUDY OF LEARNING DISABILITIES
Teachers College, Columbia University

STORY PROBLEM SOLVING: IMPLICATIONS OF RESEARCH FOR TEACHING CHILDREN WITH LEARNING DISABILITIES

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EC 140641

Technical Report #12

1980

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Abstract

A theoretical explanation of how children process story problems, and an organized series of empirical suggestions on how to teach story problem solving, has been slow to emerge in American psychological literature. Consideration of Soviet studies in mathematical instruction expands both the knowledge base and the potential instructional recommendations. In this article, the authors draw upon both Soviet and American literature to present an overview of the components of story problem solving. Factors within the individual problem solver are explored, and strategies children use in solving problems are examined. Particular attention is paid to analytic and intuitive strategies, and to how the child attempts to make the transition from concrete to symbolic problems. The most widely supported instructional principles are reviewed. The phenomenon that children who present correct oral solutions frequently err when writing the answers is examined. The article concludes with a review of how studies cited may contribute to an improved methodology for researching story problem solving.

INTRODUCTION

"Story" problems are problems that require the subject to apply mathematical knowledge (Davis & McKillip, 1980). Story problems may be presented orally or in writing, and they may be simpl· or complex. It is widely accepted now that automaticity in the basic facts is a prerequisite for success at solving story problems (Davis & McKillip, 1980; Lankford, 1974; Pincus et al., 1975; Radatz, 1979). Considerable progress has been made in describing the problem solving process, and this will be described below. For success, it is expected that the subject be able to relate to the context of the problem, that he/she be capable of analysing the key factors, and that he/she then has the ability to select and apply the appropriate mathematical operations. This specification has led some writers to offer enthusiastic remedial and instructional suggestions (e.g., Barnett, Sowder, & Vos, 1980; Davis & McKillip, 1980; Suydam, 1980). However, their enthusiasm seems to have caused them to overlook one of the major anomalies that has emerged from this research. The research upholds the appropriateness of the description of the problem solving steps, but only when oral problem solving is taking place. When the subject is required to write the solution in formal mathematical language, a whole new process appears to come into operation. If this is the case, the proffering of remedial or instructional suggestions for oral story problem solving are unlikely to be of any assistance in the creation of formal written solutions. It remains one of the challenges of the study of problem solving, to offer an explanation and remediation of this lack of transference.

The complex task of problem solving has been analysed, and a number of authors (Lester, 1980; Sowder, 1980) have identified the principal categories of variables that compose the process. Task Variables describe the factors that contribute to the difficulty of the problem itself. Subject Variables refer to individual characteristics of the problem-solver which may facilitate or retard solution, while the subject's ability to select and organize appropriate strategies is referred to as Process Variables. Instructional Variables refer to the role of instruction in enhancing or retarding the problem solving process. The present review will begin by describing the characteristics of these four categories of variables in the context of the current literature. It will then examine the literature for evidence on the reasons for lack of transference between oral and written problems. Finally, the literature will be evaluated to assess its contribution towards a refinement of the methodology employed in the study of story problem solving.

Task Variables

Problem Complexity

The nature and influence of problem complexity has been described from many different perspectives. Bana and Nelson (1977) holds the view that problem complexity comes from the distracting effect of non-mathematical information, and they suggest that to focus children's attention on the relevant operations, we should resort to the use of purely mathematical equations. Without defining complexity at all, Davis and McKillip suggest that problems can be made easier by using smaller numerical data and less elaborated stories. Barnett, Sowder and Vos (1980) suggest that the

language of the problem contributes to its complexity. They cite studies by Earp (1970) and Henny (1971) which have identified certain language factors which contribute to problem difficulty. The density of the language in the problem, the lack of contextual clues, the lack of continuity among ideas and the interspersal of numerals in the prose are all regarded as influences that can inhibit problem solving. Lester (1980) maintains that "mathematical complexity" makes problems more difficult but maintains that there is no clear evidence on the effects of readability level. Suydam (1980) agrees with Lester, that there is little evidence to support the notion that reading is as big a deterrent to problem solving as is commonly believed. Suydam also cites numerous factor-analytic studies which failed to identify any factors which accounted for a major portion of problem complexity. Carpenter (1980) suggests that success at problem solving is a function of the information processing demands of the task.

While the U.S. studies cited above all provide useful notions about problem complexity, they are nevertheless somewhat peripheral. They fail to define the components of complexity operationally, and they offer no evidence on why the child reacts adversely to these components. Soviet literature takes a different approach. Yaroschuk (1969) conducted experiments in which children were asked to solve both story problems and numerical problems. Based on his data, Yaroschuk concluded that the children were more successful at solving story problems and simple problems. The higher level of abstraction required for complex and numerical problems forced a lower level of success on the child. Yaroschuk also made the observation that in numerical problems the mathematical

structure is clear, but the subject is unfamiliar, whereas the converse is true of story problems. This observation suggests that when children are processing story problems, they may well understand the subject matter, but it is likely that they will have difficulty relating it to mathematical operations. Shchedrovitskii and Yakobson (1975) offer an interesting perspective on complexity. It is their opinion that a problem increases in complexity as its object content and the need for counting decline. They say that numerical problems in textbooks are typically difficult for children because they have no relationship to real objects, and because all of the numbers are present, requiring no counting on the part of the subject. In processing a problem, they hypothesize that the child must first visualize the problem, and then he/she must construct a mental model of the situation. Problems that lack object content interfere with the child's ability to perform these operations. In simple story problems e.g., "How many objects on the table?," the child can act directly upon the objects by counting. Shchedrovitskii and Yakobson say that as problems become more complex the child constructs a mental model and counts the objects in the model. However, problem complexity has an influence on success in solving the problem. Shchedrovitskii and Yakobson point out that children who have no difficulty solving this problem:

> A boy had 7 pencils. He lost 2. How many did he have left?

often cannot solve this one:

> A cat had some black kittens and two grey ones. Altogether there were five. How many black ones were there?

Shchedrovitskii and Yakobson hold the view that the child's increasing difficulty is caused by the increasing complexity of the problems. As the problems grow more complex, the child no longer can make parallel

models of the aggregates, and instead must have a deep understanding of the conditions of the problem, in order to select the order in which to begin counting the various numbers in the problem. Thus, Shchedrovitskii and Yakobson conclude that problem complexity is a function of problem structure.

Perhaps the person who has contributed most empirical evidence on the question of problem complexity, is Kuzmitskaya (1975). He required children to reproduce problems orally, and an analysis of their oral interpretation led him to conclude that accurate reproduction of a problem requires both comprehension and interpretation of the relations that exist in the problem. From this, Kuzmitskaya developed the hypothesis that problem complexity is related to its structure. He classified problems with direct, observable relations as simple problems, and problems with indirect relations, as complex problems. To test whether the structure of the problem, or the number of relations in the problem was the primary contributor to complexity, he gave four problems to his subjects. The problems used were simple story problems in addition and subtraction, and each was either high or low in complexity or number of elements:

1. In one cage there are 17 rabbits, and in another, 13 rabbits. Altogether, how many rabbits in the two cages? (Simple; 21 elements)

2. There were 19 plain and 6 more colored than plain pencils. How many colored pencils were there? (Complex; 18 elements)

3. There were 25 notebooks in the cupboard; 13 notebooks were distributed to the students. How many notebooks remained in the cupboard? (Simple; 21 elements)

4. A pupil spent 15 kopeks; he had 20 kopeks left. How much money had the pupil previously? (Complex; 17 elements)

Problems 1 and 3 are regarded as simple problems because they have a direct relationship, connecting the unknown to the given data. The complexity of

problems 2 and 4 is caused by the indirectness of the relationship between the present data and the unknown. Using a mentally retarded population, Kuzmitskaya found that more than twice as many students were successful with the simple problems, indicating that success at problem reproduction depends upon the character of the relations present in the problem, not on the size of the problem. Kuzmitskaya's findings on the transition from oral reproduction to written solution will be presented in a later section.

Content and Vocabulary

From the evidence presented above, it seems reasonable to state that the most important aspect of the problem content is measured in terms of the complexity of the relations present in the problem. Some authors assume that the vocabulary of a story problem is important too. Barnett et al. (1980) warn teachers about the problem of mathematical vocabulary's use of very specialized meanings. Jansson (1971) talks about the ambiguity of mathematical language and Davis and McKillip (1980) suggest presenting the problems in a simplified language. Bogolyubov (1972) says that each story problem consists of vocabulary words and operation words. The operation words are formed mathematical terms for which no substitution is possible. Without a precise understanding of the meaning of these terms, no solution is possible. The vocabulary words are readily substitutable, but Bogolyubov quotes many examples which indicate the importance of using words that are both appropriate to the context, and meaningful to the child. His studies show that when meaningfulness or appropriateness are varied, problem solution becomes difficult, if not impossible. He points out too, that particular care should be taken in differentiating closely related or visually or auditorially similar words.

Kuzmitskaya (1975) says that teachers frequently encourage children to remove the verbal or story part of the problem, to minimize distraction and to help children focus on the mathematical conditions of the problem. The result, according to Kuzmitskaya, is an increase in "nomenclature errors." This means that children use the mathematical terminology without understanding the meaning of the words. Ginsburg (1975) elaborates on this point. In one of his case studies Ginsburg discovered that one of his subjects, Patty, could solve a computational problem when it was presented using the key-word "altogether" to signify addition, but failed when the formal term "plus" was used. Ginsburg suggests that the colloquial key-word "altogether" elicited an effective informal strategy from the child, whereas "plus" elicited a formal but incorrect approach. It is clear then that "good" story problems should contain vocabulary words that are meaningful and appropriate to the situation, and that care should be taken, to ensure that formal mathematical terms are not introduced before the child is ready to accept them and to use them appropriately.

Subject Variables

As Suydam (1980) points out, it is not necessarily advantageous to point out the characteristics of good problem solvers, as ultimately, we have to deal with the general population of problem solvers. However, it is possible that an enumeration of the characteristics of good problem solvers may highlight appropriate objectives, towards which we can attempt to guide the general population of problem solvers. Kruteskii (1976) analysed the solution process of many mathematically expert problem solvers and concluded that the most important problem solving abilities were the

ability to generalize, to switch methods easily, to skip some steps, and to recall the general features of the problem, as opposed to superfluous details (Suydam, 1980). From a review of contemporary literature, Suydam identified the characteristics of good problem solvers. She says that it is reasonable to expect good problem solvers to have high IQ scores and reasoning ability, high reading comprehension scores, and high scores in quantitative and spatial abilities. Lester (1980) agrees with the characteristics identified by Suydam, but adds that a positive attitude and appropriate cognitive style are important too. By appropriate cognitive style, Lester means a high level of field-independence. Vaidya and Chansky (1980) take this argument further, by stating that not only is mathematical achievement strongly correlated with cognitive style, but that the optimal situation is where a matching of teacher and pupil cognitive styles is ensured. This argument is refuted by Saracho and Dayton (1980), as well as by the considerable evidence built up from the study of direct and indirect teaching styles (e.g., Bennett, 1976; Solomon & Kendall, 1980). The latter studies found that one teaching style (i.e., the direct style) was more highly correlated with achievement than any other, and that pupil style was irrelevant to achievement. In the context of studies in mathematics, the influence of cognitive style is strongly questioned by Fennema and Behr (1980). Fennema and Behr say that the aptitudes that are relevant to mathematical problem solving are of two types. Cognitive aptitudes describe the child's aptitudes in terms of IQ score, verbal and numerical ability, reasoning ability, and spatial visualization. Affective aptitudes refers to the positive relationship that exists between attitude and achievement, the child's level

of achievement motivation, and the possibility of sex differences. Fennema and Behr suspect that sex differences are related to the confidence/anxiety level of the student.

In dealing with the difficulties of children who are weak at story problem solving, the dichotomy described by Fennema and Behr must not be forgotten. If, as described above, the solution of complex problems depends on the perception of relations among parts, as well as certain reading and mathematical skills, it is clear that the child's level of cognitive aptitudes will affect the outcome. Perhaps even more potent though, will be the effect of affective aptitudes. If the child is beginning to fail, or has already experienced persistent failure, account must be taken of his/her level of achievement motivation, and the quality of the attitudes exhibited by the child.

Process Variables

This section is concerned with how the student organizes the approach to problem solution. The first part describes strategies employed by the problem solver, and this will be followed by an examination of the difficulties encountered by the student, in transferring from concrete to symbolic language.

Problem-Solving Strategies

Ginsburg (1976) suggests that the child's knowledge of arithmetic be described in terms of three cognitive systems which may operate concurrently, as the child solves a problem. The first system is described as natural, because it is pre-numerical, and develops outside of school. The informal system also develops prior to schooling, but it involves the

use of counting strategies, which the child has picked up incidentally. The techniques used by the child to deal with symbolic arithmetic are generally taught in school, and this is termed the <u>formal</u> system. In discussing story problem solving we are concerned with the behavior of the child who has already encountered the formal system. Therefore, it is to be expected that this child will be receiving instruction in standard algorithms for the solution of mathematical problems. Ginsburg says that typically, the teacher gives instruction on a new algorithm, and the child is then expected to master it and apply it. Ginsburg points out that although the child may "learn the algorithm," this does not imply understanding. Suydam and Dessart (1980) support the view, that to be effective, algorithms need to be taught rationally (i.e., with explanation). Ginsburg goes on to say that many children either disregard the standard algorithms in favor of "invented procedures," or else they assimilate the algorithms into existing problem solving schemes. The end result is that the child uses a unique, but intuitively rational approach to the solution of a problem. In some cases, this leads to a correct solution, but even when the final answer is incorrect, the approach can be demonstrated to be rational from the child's perspective. This has led Ginsburg to state that children's errors are rarely capricious or random, but rather that errors result from the systematic misapplication of correct procedures. This view is contrary to the earlier view of error analysts, such as for instance, Roberts (1968) or Cox (1975). Thus, it is Ginsburg's view that the child's organizing strategy is strongly influenced by (a) an understanding of the formal algorithm and (b) the invented procedures he/she devises on the basis of previous mathematical experience and an assimilation of the problem. Radatz (1979) supports Ginsburg's view that

children's errors are generally systematic and rational. Shchedrovitskii and Yakobson (1975) have arrived at a conclusion similar to Ginsburg's, from the study of the indirect story problems (i.e., problems where the unknown is not readily apparent). They state that arrival at an incorrect solution is not necessarily an indication of lack of understanding, but that it could be caused by the child selecting an inappropriate, but to the child, intuitively acceptable technique. They say that in their experiments, children often understood the requirements of the problem, and could state the solution orally. However, when it came to the selection and employment of formal mathematical operations, they made errors. Shchedrovitskii and Yakobson suggest that this may have occurred because the children's understanding relates to their own intuitive problem solving methods (cf. Ginsburg's "invented procedures"), and not to the socially fixed mathematical methods used by adults. Like Ginsburg, Shchedrovitskii and Yakobson emphasize the developmental aspects of problem solving. They say that the child's strategy is very much a function of how he/she assimilates formal algorithms into his/her personal solution system. Shchedrovitskii and Yakobson point out that the logical outcome of this is that we should be attentive to the appropriate presentation of algorithms to young children, as these techniques are apt to be an enduring part of their scheme.

Mikhalskii (1975) looks at another aspect of the student's strategy. He believes that a preliminary analysis of the conditions of the problem is essential to successful solution. Mikhalskii says that the conditions of the problem consist of:

a) the requirement or question of the problem.
b) the data given in the problem.
c) the relationships within the given data.

Analysis of these conditions leads to comprehension and thus enables the student to select the appropriate arithmetical operations to solve the problem.

In effect, according to Mikhalskii, this approach causes the problem solver to decompose a complex problem into its component parts. To test the importance of preliminary analysis, Mikhalskii carried out a study which compared the effectiveness of analysis, synthesis, and solution by analogy as problem solving strategies. To his surprise, Mikhalskii discovered that analysis was not a differentially more effective technique than either of the others. It is clear, both from Mikhalskii's work, and from the studies of Kuzimitskaya (1975), that training in analysis improves oral reproduction of the problem. The fact that this has little bearing on the final solution, will be discussed in a later section. Mikhalskii's analysis revealed some interesting information about the strategies children employ in problem analysis. He found that in describing the solution orally, the most common method employed was the synthetic method. The students carried out the calculations first, and only then addressed themselves to the question posed in the problem. When the experimenter attempted to focus their attention on the question, the children usually stated the question without referring to the conditions of the problem. Furthermore, young children tended to perseverate or repeat the solutions from an earlier problem, in a stereotyped manner. Older children were able to solve subparts of the problem without understanding the overall question. Mikhalskii concludes then that considerable time should be devoted to techniques which help the child analyse the problem question and decompose its subparts. This view is supported widely in the literature (Bogolyubov, 1972; Schoen & Oehmke, 1980).

In summarizing the literature on problem solving strategies, two main themes emerge. Firstly, it is clear that attention must be given to decomposition of the problem question, to focus the child's mind on the conditions of the problem. Once this has been achieved, account must be taken of the unique strategies each child employs. If, as has been suggested,

these strategies are intuitively rationsl, the basis of this rationality must be examined for each child, before remediation can be suggested.

Concrete-Symbolic Transition

Many authors have shown concern over the difficulties children have in transferring from concrete to symbolic problem solving, and their remedial suggestions will be examined in a later section. Two of the Soviet authors offer some guidelines as to how this transition occurs. Davydov (1975) describes a sequence of number acquisition that is similar to the Gelman (1978) model. This view proposes that as the child moves away from concrete counting, the real objects are replaced first by hand movements, and later by accentuated vocal stress while counting. Davydov believes that with maturity, this process continues in an internalized manner. Shchedrovitskii and Yakobson (1975) specify the parameters of just such a mental model. They believe that as the child encounters a new problem situation, he/she first resorts to the counting skills in his/her present repertoire. If these are inadequate to the task the child is forced to resort to a new strategy. Shchedrovitskii and Yakobson say that this strategy is modeling. At first the child models by using substitute objects (e.g., his/her fingers), in a one to one correspondence with the numbers in the problem. Later, the child learns to construct mental models, and ultimately to keep track of the separate aggregates when solving "indirect" problems. The construction of models, in their opinion, depends on more than just a simple visualization of the problem. It requires a detailed analysis of the conditions of the problem. Modeling actually requires a series of transformations and substitutions, until ultimately, the child arrives at an economic representation of the problem. Shchedrovitskii and Yakobson believe that the ability to construct mental

models is not automatically acquired, but should be the subject of direct instruction.

Instructional Variables

Evidence for the type of teaching style that is most highly correlated with student achievement has come from a variety of sources. Carroll (1963) pointed out that student achievement is highly related to opportunity to learn, or academic engaged time. Carroll stated that the longer the teacher spends in actual teaching activities, as opposed to managerial tasks or behavioral or procedural interventions, the higher will student achievement be. Recent studies have provided supportive evidence for this view (Gettinger & White, 1979, 1980; Rosenshine & Berliner, 1978). Kounin (1970) identified a number of teacher managing behaviors that were highly correlated with pupil achievement. Broadly, Kounin stated that teachers who were alert, who created smooth transitions between activities and who monitored seat work carefully, had higher achieving pupils. Kounin's work has been corroborated, too, by a number of other researchers (Anderson, Evertson & Brophy, 1978; Brophy & Evertson, 1976; Good & Grouws 1979). In a review of teacher-behavior studies, Brophy (1979) says that teacher talk in the form of lecturing or demonstrations, is highly correlated with student achievement. If all these separate views are combined with the widely recognized direct learning principles of presenting the material in small units, periodic review, drill and practice and teaching to mastery (Bryant, Note 1) then a picture of what is known as the "direct teacher" emerges.

The direct teacher is regarded as having firm, directive control over the class. He/She talks a lot, indulging in much lecturing and demonstration,

and offering pupils only a minimal amount of opportunity to interact together. This type of teacher is concerned with the direct communication of knowledge, not with leading children towards discovery. Some recent, large field studies have provided convincing evidence that this type of teacher produces greater student achievement. Bennett (1976) clearly demonstrated the greater effectiveness of formal teachers than informal ones. Solomon and Kendall (1979) showed that teacher directiveness was the factor most associated with pupil achievement. (For extensive reviews, see Brophy, 1979; Dunkin & Biddle, 1974; and Rosenshine & Furst, 1973.)

Thus, in looking at the teaching of story problems, it is reasonable to state that direct teaching is likely to be more effective in producing learning. It remains then, to look at the story problem solving literature in order to establish what material might most appropriately be taught. Studies which were based on error analysis suggested that instruction should concern itself with remediation of specific errors (Cox, 1975; Pincus et al., 1975; Roberts, 1968; Smith & Lovitt, 1973). However, with interest increasingly centering on the problem solver's processes, instructional recommendations focus on this area too.

Yaroschuk (1969) says that we should teach children how to relate the concepts in the story to the required mathematical operations. He does not specify how, but Shchedrovitskii and Yakobson (1975) suggest that it is best done by instruction in model making. This instruction would require teaching the problem solver how to move gradually from physical representations to symbolic representations. Kuzmitskaya (1975) is also interested in the broad area of problem analysis. His studies clearly showed the importance of comprehension and interpretation, and he says that teachers

should focus their instruction on these skills. Both Kuzmitskaya (1975) and Mikhalskii (1975) illustrated the importance of this training in problem analysis, in order to produce correct oral reproductions. They suggest that great attention should be given to training in problem analysis and problem decomposition. Kalmykova (1972) gives some specific suggestions on how these skills may be taught. She says that the pupil should be encouraged to read the problem for meaning. Then, each datum should be analyzed, and help should be given in the differentiation of confusable concepts. The pupils should be required to justify the selected method of solution and they should be required to state the question. After the problem has been solved, they should check their work and analyze their errors. Many other authors support these suggestions (Bogolyubov, 1972; Davis & McKillip, 1980; Radatz, 1979; Schoen & Oehmke, 1980). With regard to the terminology of story-problems, Bogolyubov says that mathematical terminology should be taught without amgibuity, and that other vocabulary words then be substituted to provide practice in generalizing problem solutions. Further suggestions about how to aid with generalization come from Mikhalskii (1975), and are substantially supported by Suydam (1980). Mikhalskii suggests that practice in solving complex problems could be easily acquired by having children solve two simple problems first, and then combine them to make one complex problem. He also suggests selecting problems with no numbers. The children would be required to select different series of numbers and to derive appropriate solutions. Ginsburg's (1976) studies suggest the importance of the child's informal language. As a result, the introduction of formal terms and symbols should be made in a clear and non-ambiguous manner. Ginsburg's work

emphasizes that algorithms should be taught rationally, with emphasis given to the application of the algorithm to problems.

Oral Versus Written Solutions

Shchedrovitskii and Yakobson (1975) credit Ern (1915) with the discovery that children who could verbalize problem solutions, frequently wrote out the solutions incorrectly. From their studies Shchedrovitskii and Yakobson concluded that this failure to write out the solution correctly did not stem from a lack of understanding. They say that the children solved the problems mentally by using "invented procedures" (Ginsburg), but that in writing out the formal solution, they "surrendered to the logic of what they had written" (p. 68). Shchedrovitskii and Yakobson make no attempt to explain this phenomenon, rather, they concentrate on an examination of the child's informal processes. While Mikhalskii's (1975) research does not provide an explanation either, his work adds some relevant information. Mikhalskii found that children could be trained to solve problems using analysis, but that the techniques were abandoned by the students when it came to writing out the solutions. At this point, the students either repeated the solution to an earlier problem in the set, in a stereotypical manner, or else they solved the problems by drawing upon analogies stored in memory. In fact Mikhalskii found that when a child is confronted with the necessity for a written solution, all of his/her energy goes into recalling similar solutions or stereotyped patterns from the past, not into problem analysis. Mikhalskii studied a retarded population, and the other important conclusion he came to was that they are preoccupied with reproductive thought. By reproductive thought in

this context is meant the recollection of previous experience as opposed to logical analysis. Mikhalskii does not offer a hypothesis as to the process involved here, but he suggests an explanation, one that is later echoed by Suydam (1980). Mikhalskii thinks that this occurs because the child, and in particular the retarded child, has a tendency to seek a solution. This orientation is so strong that Mikhalskii describes it as "the attempt not to reason, but just to operate; not to substantiate or explain one's actions, but to solve the problem" (p. 83, emphasis added). The pupils concentrated on "How can I solve the problem," rather than asking themselves what had to be solved, and what might be the most appropriate operations to use. Mikhalskii also makes the point that for non-retarded children, the problem may be developmental. He found that with increasing maturity, children were less likely to respond stereotypically, they were less stubborn in holding onto their initial response, and they were increasingly governed by the logic of the operations and conditions of the problem.

Kuzmitskaya (1975) also discovered that mentally retarded students had a tendency to respond stereotypically, by drawing upon the most recent solution in their minds to solve a subsequent set of problems. Kuzmitskaya does not deal with the oral-written problem, but in his analysis of oral reproductions, he found many inaccuracies in the students' interpretations of the original problem. He attributes these discrepancies in reproduction to difficulties in the interaction of the signal system of the subjects. To support this position, Kuzmitskaya says that children who could reproduce direct problems without difficulty, had real problems with indirect ones. He says that this occured because indirect problems have much less object content and so the students have difficulty representing them by visual

images. This suggestion of discontinuity between systems is interesting in the light of Ginsburg's (1976) work. Ginsburg found out that students who could calculate at ease, orally, had real problems when they were required to write the solution. He says that children have difficulty with written work because it involves a set of symbols that are very discrepant from their own "invented procedures." Children need to be taught the meaning of these symbols, and how they integrate with the children's previous knowledge. Ginsburg says that understanding, and therefore correct solutions, can only come through the integration of systems. It is Ginsburg's thesis that particularly children who are in any way mathematically disadvantaged, can only be taught through the integration of new knowledge with their existing informal repertoire of experience.

Both the suggestions of Ginsburg and of the Soviet authors are tentative hypotheses, based on limited data from specialized populations. At the moment though, they appear to provide the only explanation of the discrepancy between informal and formal performance at mathematical problem solving tasks.

Methodological Implications

Shchedrovitskii and Yakobson (1975) believe that people try to solve new problems from within their existing repertoire of solutions. In order to examine their problem solving processes it is necessary to expose them to more difficult problems, so that the pressure to find solutions will force them to externalize the processes they are employing.

Mikhalskii's technique of providing four categories of problems across grade levels is useful. Mikhalskii selected ordinary classroom

story-problems to examine pupils' existing strategies. He also used three other problem types (problems with missing numbers, problems requiring the student to state the question, and problems requiring analysis). These problems represented a new format to the students, and Mikhalskii says that they worked admirably in revealing differences in strategy across students, differences which were not visible using regular solution strategies. In terms of instructions, Mikhalskii simply instructed children to solve the problems orally. Mikhalskii also provides many examples of how to question a student, in order to elicit either synthetic or analytic problem solving. Bogolyulbov (1972) says that student-generated problems are useful for eliciting problem solving strategy, and for testing problem solving competence. This technique is also commended by Mikhalskii (1975), Radatz (1979), and Shchedrovitskii and Yakobson (1975).

Kuzmitskaya, too, has used interesting techniques. He succeeded in controlling for problem size by limiting the number of elements in each story problem. He tested oral reproduction of problems, by presenting the problems on cards, and asking the subjects to reproduce them orally. In some of his studies, Kuzmitskaya presented the children with both direct and indirect problems, in order to examine the effect of problem complexity. He did this by contrasting story problems with direct relationships with other problems where the relationships were not so readily visible. Kuzmitskaya's discoveries on nomenclature, and the effect of eliminating object content, are easily studied by contrasting story-problems with mathematically similar numerical equations. In his studies, Kuzmitskaya not only analysed oral reproductions but he also studied their effect on subsequent problem solution.

While the Soviet authors have studied both oral reproductions and

the protocols resulting from guided interviews, the only person to attempt to legitimize this as a methodology has been Ginsburg (1976). Ginsburg says that we need a procedure which will provide an account of the child's processing while he is solving problems. This procedure must be able to describe the subject's strategies, whether they are successful or result in error. Ginsburg justifies using a non-standardized instrument, on the grounds that standardized tests are too rigid and complex. An added justification is that flexible instruments are more likely to tap competence, not just mere performance. Ginsburg gives an extensive description of how the clinical interview method may be used to do individual cognitive case studies. In effect, these case studies are extensive interview sessions, in which the interviewer tests and alters hypotheses in light of the incoming data.

In summary, both the studies of the Soviet authors, and those of Ginsburg have much in common. Both are concerned with an examination of the active processing of the child. Both are quite fruitful in terms of offering testable hypotheses, and both offer promise for theory construction. However, both labor under the difficulties of using techniques that have a high response cost. Furthermore, as Ginsburg points out, drawing generalizable conclusions from these techniques is somewhat worrying, until such time as empirical data on instrument reliability has been collected.

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